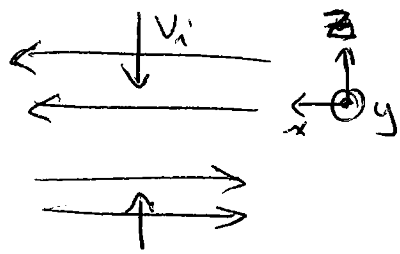
(6.5 a i) (6.14) gives $B_x = \pm B_0\left(1 - e^{\mp\mu_0\sigma V_i z}\right)$ $\qquad z \gtrless 0$

To calculate $\underset{\sim}{E}$ recall that $\underset{\sim}{E} + \underset{\sim}{V}\times\underset{\sim}{B} = \underset{\sim}{J}/\sigma$



so $\underset{\sim}{E} = -\underset{\sim}{V}\times\underset{\sim}{B} + \underset{\sim}{J}/\sigma$

Now $\underset{\sim}{V} = \mp V_i \hat{z}$

while $\mu_0 \underset{\sim}{J} = \nabla\times\underset{\sim}{B} = \left(\hat{z}\frac{d}{dz}\right)\times\left(B_x\hat{x}\right) = \hat{y}\,\frac{dB_x}{dz} = \hat{y}\,(\pm B_0)(-)(\mp\mu_0\sigma V_i)\,e^{\mp\cdots}$

So $\frac{\underset{\sim}{J}}{\sigma} = \frac{1}{\cancel{\mu_0\sigma}} B_0 \cancel{\mu_0\sigma} V_i\, e^{\mp\mu_0\sigma V_i z} = V_i B_0 e^{\mp\mu_0\sigma V_i z}\hat{y}$

Thus $\underset{\sim}{E} = -(\mp V_i\hat{z})\times\left[\pm B_0\left(1 - e^{\mp\mu_0\sigma V_i z}\right)\hat{x}\right] + V_i B_0 e^{\mp\mu_0\sigma V_i z}\hat{y}$

$\qquad = + V_i B_0 \hat{y}\left(1 - e^{\mp\mu_0\sigma V_i z}\right) + V_i B_0 e^{\mp\mu_0\sigma V_i z}\hat{y} = \underline{\underline{V_i B_0 \hat{y}}}$

Note $\underset{\sim}{E}$ = const as required by $\nabla\times\underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t} = \underset{\sim}{0}$

Poynting Flux $\underset{\sim}{S} = \frac{\underset{\sim}{E}\times\underset{\sim}{B}}{\mu_0} = \frac{V_i B_0}{\mu_0}\,\hat{y}\times(\pm B_0)\left(1 - e^{\mp\mu_0\sigma V_i z}\right)\hat{x}$

ie $\underset{\sim}{S} = \frac{V_i B_0^2}{\mu_0}(\mp\hat{z})\left(1 - e^{\mp\mu_0\sigma V_i z}\right) \qquad z \gtrless 0 \Rightarrow$ toward $z=0$

Now $|\underset{\sim}{S}(z\to+\infty)| = \underline{\underline{V_i B_0^2/\mu_0}}$ while

$\int_0^\infty \underset{\sim}{E}\cdot\underset{\sim}{J}\,dz = \int_0^\infty V_i B_0\,\hat{y}\cdot\sigma V_i B_0 e^{-\mu_0\sigma V_i z}\hat{y}\,dz = V_i^2 B_0^2\sigma\int_0^\infty e^{-\mu_0\sigma V_i z}\,dz$

$\qquad = V_i^2 B_0^2 \sigma \frac{1}{-\mu_0\sigma V_i}\left[e^{-\mu_0\sigma V_i z}\right]_0^\infty = \frac{V_i B_0^2}{-\mu_0}(0-1) = \underline{\underline{\frac{V_i B_0^2}{\mu_0}}}$

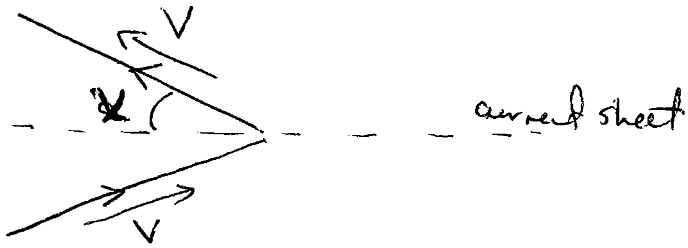
so $|\underset{\sim}{S}(z\to+\infty)| = \int_0^\infty \underset{\sim}{E}\cdot\underset{\sim}{J}\,dz$

$\underset{\sim}{S}(z\to+\infty)$ is the Poynting Flux of energy ~~[illegible]~~ headed toward sheet

$\int_0^\infty \underset{\sim}{E}\cdot\underset{\sim}{J}\,dz$ = total energy dissipated due to "Joule heating"
(in electrical terms it's Voltage x Current)

So result says all energy coming in from $z\to+\infty$ gets dissipated in region $0<z<+\infty$.

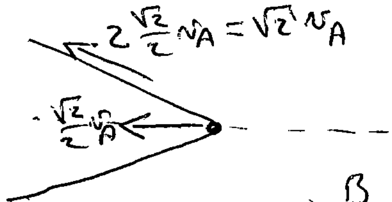
6.5.2



a) In frame in which field lines at rest particles don't change energy ($\underline{E}=0$). Each particle changes momentum by an amount $2mV\cos\alpha \approx 2mV$ on traversing sheet. Now the flux coming in along a given field line is nV thus the total rate of change of momentum per unit time per volume is $(nV)(2mV) = 2mnV^2$ = field line tension $= \frac{B^2}{\mu_0}$

whence $V^2 = \frac{1}{2}\frac{B^2}{\mu_0 m n} = \frac{1}{2}\frac{B^2}{\mu_0 \rho} = \frac{1}{2} v_A^2 \Rightarrow \boxed{V = \frac{\sqrt{2}}{2} v_A}$

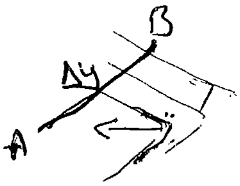
b) Transforming to observer's frame is essentially at speed V as given in (a) ~~...~~ so pt of intersection moves at speed $V = \frac{\sqrt{2}}{2} v_A$. In this frame ΔV of particles is $2V$ so they outflow at $\underline{2V = \sqrt{2}\, v_A}$



c) In time t flux crossing line AB is

$$\Phi = \Delta y\left(\frac{\sqrt{2}}{2} v_A t\right) B_z \quad \text{so for } \Delta y = 1, t = 1$$

$$\Phi_B = \frac{\sqrt{2}}{2} v_A B_z$$



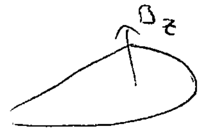
we can find E_y at current sheet (where $B_x \to 0$) since

$$\frac{E_y B_z}{B_z^2} = V = \frac{\sqrt{2}}{2} v_A \Rightarrow \underline{E_y = \frac{\sqrt{2}}{2} v_A B_z = \Phi_B}$$

It is apparent that Φ_B depends on total field (which is dominated by B_x & doesn't change outside layer) and B_z which is assumed small but constant. Note $B_z \to 0 \Rightarrow$ Reconnection $\to 0$

d) Back in field line rest frame within current sheet
$B_x = 0$ + particle has speed V in $\hat{x}$ initially (ie ⊥r $\underset{\sim}{B}$).

So $r_L = \frac{m v_\perp}{qB} = \frac{mV}{qB_z}$. To reverse direction



requires $\frac{1}{2}$ circle, ie $\Delta y = 2r_L = \frac{2m}{qB_z}\frac{\sqrt{2}}{2}v_A$

e) Then energy gain is $qE_y\Delta y = \cancel{q}\,\cancel{\frac{\sqrt{2}}{2}}v_A\cancel{B_z}\,\frac{\cancel{2}m}{\cancel{qB_z}}\,\cancel{\frac{\sqrt{2}}{2}}v_A = mv_A^2$
(in observers' frame)

But energy gain $= \frac{1}{2}mV_{out}^2 = mv_A^2 \Rightarrow \underline{\underline{V_{out} = \sqrt{2}\,v_A}}$