Formula Sheet (in units $\hbar = c = 1$)

4-vector notation:

$$a \cdot b = a^\mu b_\mu = a_\mu b^\mu = a^\mu b^\nu g_{\mu\nu} = a_\mu b_\nu g^{\mu\nu} \text{ with } g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$x^\mu = (t, \vec{x}) \ , \ x_\mu = (t, -\vec{x})$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right) \ , \ \partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right) \ , \ \widehat{p}^\mu = i\partial^\mu \ , \ \widehat{p}_\mu = i\partial_\mu$$

Klein-Gordon equation: $(-\widehat{p} \cdot \widehat{p} + m^2)\psi = (\partial_\mu \partial^\mu + m^2)\psi = (\Box + m^2)\psi = 0$

Free Dirac equation in Hamiltonian form: $i\frac{\partial}{\partial t}\Psi = (\vec{\alpha} \cdot \widehat{\vec{p}} + \beta m)\Psi$, or in covariant form:

$$(i\partial\!\!\!/ - m)\Psi = (i\gamma^\mu \partial_\mu - m)\Psi = (\widehat{p}\!\!\!/ - m)\Psi = (\gamma \cdot \widehat{p} - m)\Psi = (\gamma^\mu \widehat{p}_\mu - m)\Psi = 0$$

Dirac and Gamma matrices:

$$(\alpha^i)^2 = \mathbb{I}\, , \ i = 1,2,3; \ \beta^2 = \mathbb{I}; \ \alpha^i\alpha^j + \alpha^j\alpha^i = 0\, , \ i \neq j; \ \alpha^i\beta + \beta\alpha^i = 0\, , \ i \neq j;$$

$$\gamma^0 = \beta, \ \gamma^i = \beta\alpha^i, \ \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{I}\, ,$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

Dirac representation:

$$\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \ i = 1,2,3 \ , \ \beta = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix},$$

where the Pauli matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Note that α^i, β and γ^0 are Hermitian, wheras the γ^i are anti-Hermitian.