2 Lecture 2

2.1 Disc Formation *via* Cloud Collapse

The collapse of a cold cloud with no angular momentum (*i.e.* $\Omega = 0$) would lead to the formation of a compact, spherical star. If $\Omega \neq 0$, however, then a protostellar disc is formed. Such discs are now commonly observed in star forming regions of the Galaxy, e.g. HST images of the propylids in the Orion nebula.

One can estimate the size of the disc that will be formed by considering the conservation of angular momentum during the collapse of a protostellar cloud to form the disc.

A gas cloud of radius R_c is initially rotating with an angular velocity Ω. The angular momentum about the centre of the cloud of an element of gas of mass δm at the edge is

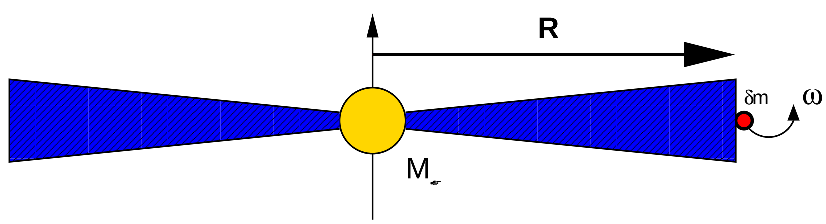
$$J \;=\; \delta m \, R_c^2 \, \Omega \;\; .$$

The specific angular momentum is the angular momentum per unit mass, and is therefore

$$j \;=\; J/\delta m \;=\; R_c^2 \, \Omega \qquad (40)$$

for the gas at the edge of the cloud.

The cloud collapses under its own gravity to form a rotating protostellar disc.



Let ω be the angular velocity of the gas at the edge of the disc, so that from equation (1)

$$\omega^2 \;=\; \frac{GM}{R^3} \;\; ,$$

for a circular orbit about a central protostar of mass M. Consider a fluid element at the disc edge with mass δm. The angular momentum of this element is

$$J \;=\; \delta m \, R^2 \, \omega \;\; . \qquad (41)$$

The specific angular momentum is therefore

$$j \;=\; \frac{J}{\delta m} \;=\; R^2 \, \omega \;=\; \sqrt{GMR} \;\; , \qquad (42)$$

for Keplerian motion.

If the angular momentum is conserved during the collapse,

$$R_c^2\,\Omega \;=\; \sqrt{GMR_{disc}}$$

from equations (40) and (42). The radius of the disc therefore may be written

$$R_{disc} \;=\; \frac{j^2}{GM}. \tag{43}$$

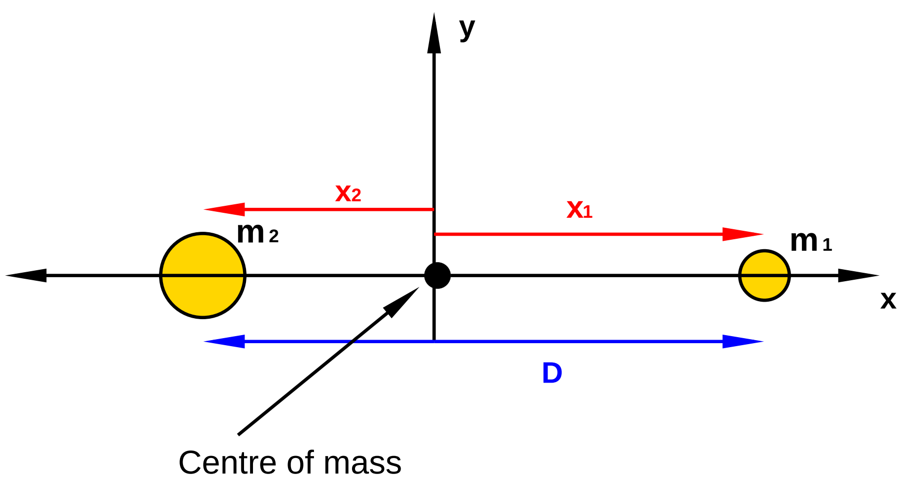
If we apply equation (43) to the collapse of a rotating cloud, then a cloud of mass $M \simeq 1\ \mathrm{M}_\odot \simeq 2 \times 10^{30}\,\mathrm{kg}$ is typically observed to have a radius of $R_c \simeq 0.1\ \mathrm{pc} \simeq 3 \times 10^{15}\,\mathrm{m}$ and an angular velocity $\Omega \simeq 10^{-14}\ \mathrm{rad\,s^{-1}}$, giving $j \simeq 10^{17}\ \mathrm{rad\ m^2\,s^{-1}}$. Thus the predicted radius of a protostellar disc forming from such a cloud will be $R_{disc} \simeq 6 \times 10^{13}\ \mathrm{m} \simeq 400\ \mathrm{AU}$. The collapse time τ_c for the cloud is given approximately by the free-fall time τ_{ff},

$$\tau_c \;\simeq\; \tau_{ff} \;=\; \pi\sqrt{\frac{R_c^3}{8GM}} \;\simeq\; 5 \times 10^5\ \ \mathrm{yr} \tag{44}$$

if the initial cloud radius $R_c \simeq 0.1$ pc.

2.2 Disc Formation in Close Binary Systems

Here we introduce the concept of the Roche potential for binary stars in circular orbit.



We will work in a rotating reference frame based on the centre of mass of the binary system. We will assume that the stars are located on the x–axis in this rotating coordinate system. The stars have masses m_1 and m_2, respectively, and have positions x_1 and x_2 given by

$$x_1 = \frac{Dm_2}{(m_1 + m_2)}, \quad x_2 = -\frac{Dm_1}{(m_1 + m_2)} \tag{45}$$

where $D =$ the separation between the stars given by $D = x_1 - x_2$. The frame rotates at the orbital angular velocity of the binary system given by

$$\Omega^2 = \frac{G(m_1 + m_2)}{D^3}. \tag{46}$$

We will consider a general point P with coordinates $(x,\ y)$ in this reference frame and calculate the gravitational and centrifugal potential at this position.

The gravitational potential due to the two stars is

$$\Phi_{grav}(x, y) \ = \ - \frac{Gm_2}{\sqrt{y^2 + (x - x_2)^2}} \ - \ \frac{Gm_1}{\sqrt{y^2 + (x - x_1)^2}} \ . \tag{47}$$

The centrifugal potential is

$$\Phi_{cent}(x, y) = -\frac{1}{2}\Omega^2(x^2 + y^2) \tag{48}$$

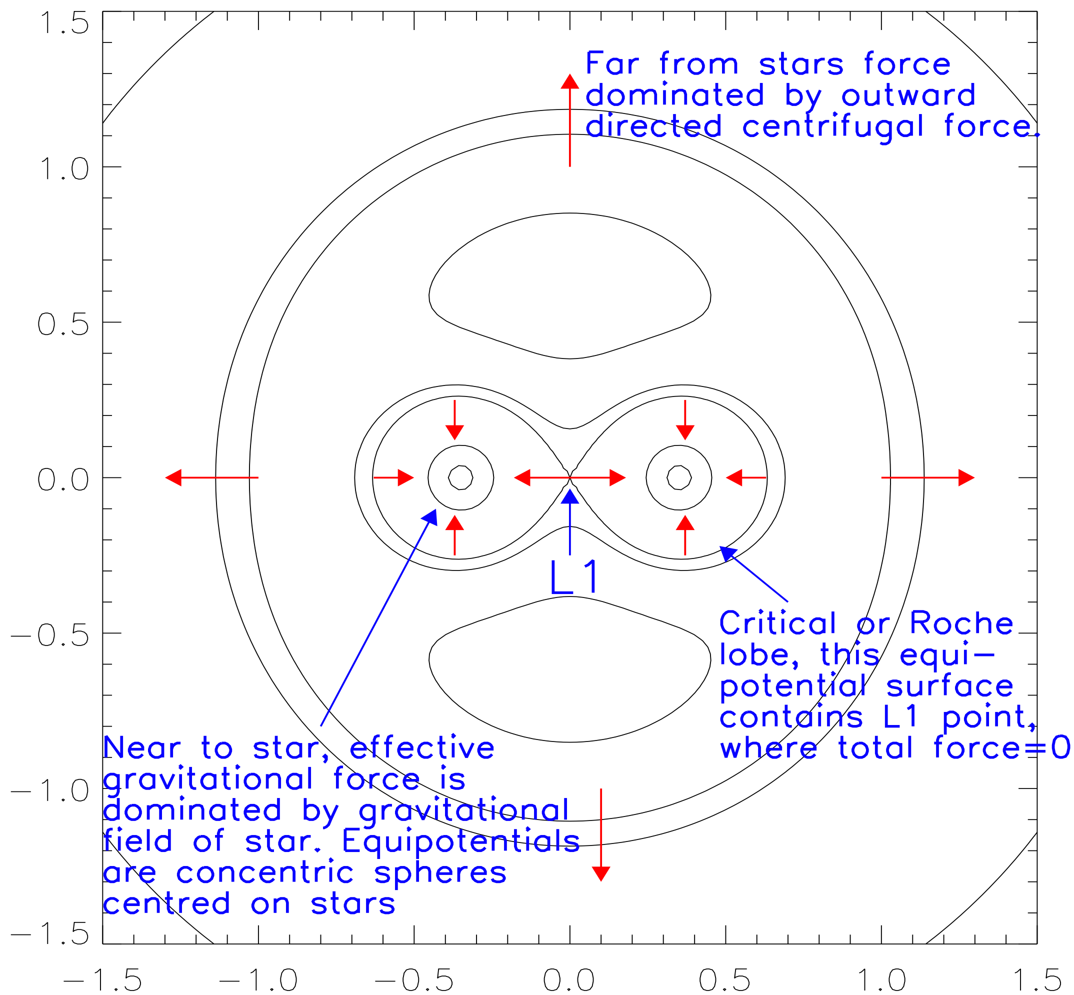
since the centrifugal acceleration is given by $-\boldsymbol{\nabla}\Phi_{cent} = \Omega^2\, r$.

The total potential is then

$$\Phi(x, y) \ = \ - \frac{Gm_2}{\sqrt{y^2 + (x - x_2)^2}} \ - \ \frac{Gm_1}{\sqrt{y^2 + (x - x_1)^2}} \ - \ \frac{1}{2}\,\Omega^2\,(x^2 + y^2) \ . \tag{49}$$

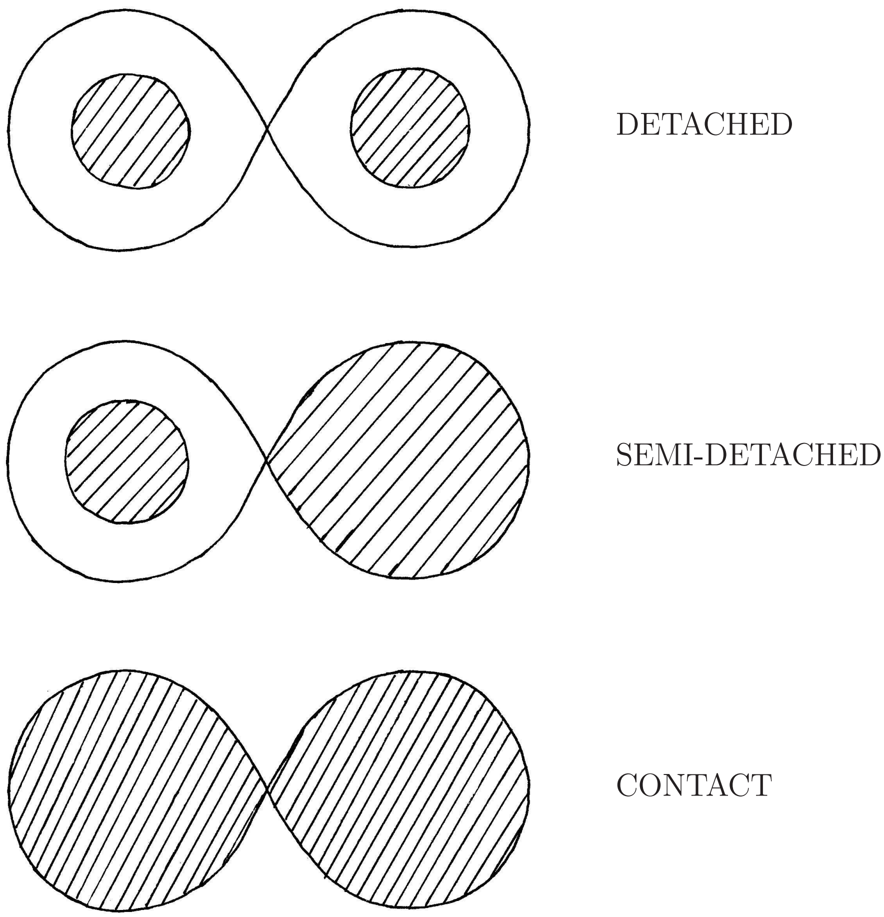
For a test particle placed in this rotating frame to be in equilibrium (*i.e.* no forces acting on it), $\boldsymbol{\nabla}\Phi = 0$. Positions where $\boldsymbol{\nabla}\Phi = 0$ are known as Lagrange points, and there are five such points all together which are usually labelled as L_1 ... L_5. The diagram on page 16 shows the most important Lagrange point for the case of close binary stars, namely the L_1 point.

We can consider the equipotential surfaces (also known as Roche equipotentials).



2.2.1 Semi-detached Binary Star Systems with Compact Components

Classification of close binaries with respect to the critical lobe:



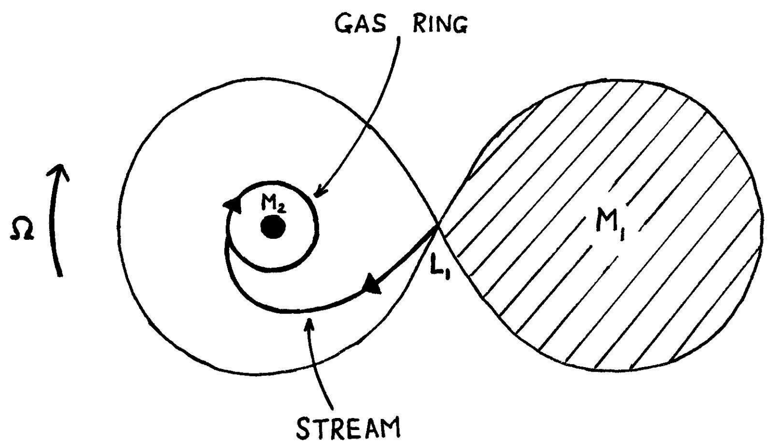
In this course we will primarily focus on semi–detached systems since they are the ones in which discs are likely to form *via* the process of Roche lobe overflow. Disc formation can only occur if the detached component of these systems is fairly compact, for reasons that are discussed later. The different types of semi–detached systems often found in astrophysics are listed in table 1.

The Roche–lobe filling component can fill its critical lobe for two basic reasons. First, the star may expand because of stellar evolution. Second, orbit shrinkage may occur because of angular momentum loss due to stellar winds, tides, or gravitational radiation, with the result being that the critical lobe itself shrinks in size until it is filled by the star. Both of these basic scenarios lead to mass transfer through the L1 point, so that mass is transferred from the donor star to the compact companion.

The gas stream from the L1 point goes into a ring about m_2, since the material cannot fall directly onto m_2 by virtue of its angular momentum. One can estimate the size of the ring formed by consideration of the conservation of angular momentum.

Name	Compact Component	Lobe Filling Component
Cataclysmic Variable (C.V.) (Novae) (Recurrent Novae) (Dwarf Novae)	White dwarf	Low mass main sequence star
Low Mass X-ray Binary	Neutron star or Black hole	Low mass main sequence star
X-ray Binary	Neutron star or Black hole	Early-type massive star

Table 1: types of interacting binary stars.



Let the distance between the L1 point and m_2 be a. The specific angular momentum of material at L1 relative to m_2 is $j = a^2\Omega$. If material flowing through the L1 point goes into circular orbit about m_2 with radius R, then $j = \sqrt{Gm_2R}$ which follows from equation (42). Equating these expressions leads to

$$a^4\Omega^2 = Gm_2R \tag{50}$$

But since $\Omega^2 = G(m_1 + m_2)/D^3$ we find that

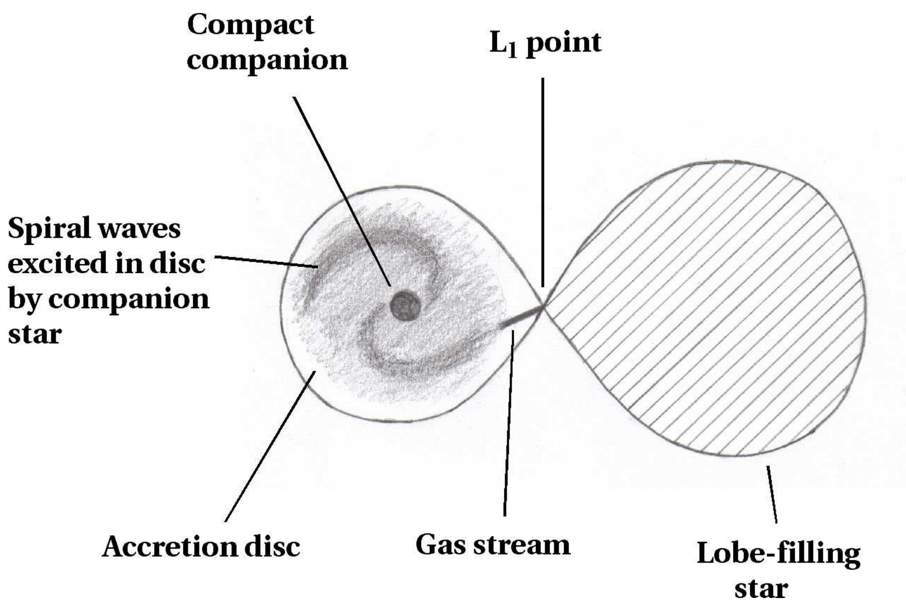
$$R = \frac{(m_1 + m_2)}{m_2}\frac{a^4}{D^3} \tag{51}$$

For $m_1 = m_2$ (*i.e.* an equal mass binary system), $a = D/2$ and

$$R = \frac{D}{8} .$$

This illustrates the point that the mass-receiving component of the binary system *must be quite compact* in order for a ring to form. So we expect that interacting binary stars that have an accretion disc will have a compact object, such as a white dwarf, a neutron star or a black hole.

If angular momentum was conserved, material would stay in a ring without expanding or accreting onto m_2. In actual fact, discs are observed to expand to fill their Roche lobes and contain spiral structure.
For example, IP Peg (a CV system) has been observed using the technique of Doppler tomography which reveals the apparent existence of spiral density waves. (Note the abbreviation CV to mean a *cataclysmic variable* star system.)



Accretion discs in close binary systems expand until they are truncated close to the Roche lobe by the gravitational forces/torques due to the companion star m_1. These forces are also responsible for generating the spiral density waves.

The fact that discs only form when the accreting star is a compact object leads to the expectation that if the lobe filling star is a main sequence star of mass $\simeq 1\ \mathrm{M}_\odot$, then the compact object must be a white dwarf (with radius $\sim 1\ \mathrm{R}_{Earth}$), neutron star (radius ~ 10 km), or a black hole (radius of event horizon ~ 3 km if $M = 1\ \mathrm{M}_\odot$).

Expansion of the gas ring to form a lobe–filling disc requires angular momentum transport to occur. The outer parts of the ring/disc must gain angular momentum in order to move outwards, and the inner parts must lose it in order to move inwards and accrete onto the compact object.