6 Week 9

6.1 T Tauri Discs and the Minimum Mass Solar Nebula

T Tauri Discs: As we have seen in the accompanying power point slides, T Tauri stars are very young, pre-main-sequence objects that show mass loss through winds and also show accretion from surrounding discs. Observations indicate that T Tauri discs have masses in the range 10^{-3} – 10^{-1} $M_\odot$, and have accretion rates between $\dot{m} \sim 10^{-9}$ $M_\odot$ yr^{-1} – 10^{-7} $M_\odot$ yr^{-1}. This leads to expected lifetimes in the range 10^6 – 10^7 yr, in basic agreement with observations.

Mass distribution in the Solar System and Minimum Mass Solar Nebula: Models of the protoplanetary disc that was the precursor to our own Solar System often assume a surface density distribution of $\Sigma \propto R^{-3/2}$, which originates from the current mass distribution of the planets as a function of their orbital radius. The surface density of the minimum mass solar nebula model obeys

$$\Sigma = 1700 \left(\frac{R}{1AU} \right)^{-3/2} \quad \mathrm{gcm}^{-2}. \tag{134}$$

If $\Sigma \propto R^{-3/2}$, then the disc mass as a function of radius scales as $M_D(R) \propto R^{1/2}$. For example, for $\Sigma = \Sigma_0 R^{-3/2}$ we have

$$M_D(R) \;=\; \int_0^R 2\pi R \Sigma_0 R^{-3/2}\, \mathrm{d}R \;=\; 4\pi\Sigma_0 R^{1/2} \ .$$

The surface density may the be written as

$$\Sigma(R) \;=\; \Sigma_0 R^{-3/2} \;=\; \frac{M_D(R)}{4\pi R^2} \ . \tag{135}$$

If we take a disc containing 10^{-2} $M_\odot$ and of radius ~ 40 AU, then at 5 AU

$$M_D(5\mathrm{AU}) \;=\; 10^{-2}\sqrt{\frac{5}{40}} \;\sim\; 3.5\ \mathrm{M}_J$$

where M_J = Jupiter's mass = $0.0010 M_\odot$.

T Tauri Discs compared with Minimum Mass Solar Nebula: To within order of magnitude, T Tauri discs have properties similar to the minimum mass solar nebula. Using equation (135), we find that the surface density at 5 AU in the minimum mass solar nebula is $\Sigma \sim 160\,\mathrm{g\,cm^{-2}} = 1600\,\mathrm{kg\,m^{-2}}$). Temperatures are $T \sim 100$ K at 5 AU, the mean molecular weight is $\mu \sim 2$, and so $H/R \simeq c_s/(R\Omega) \sim 0.04 – 0.05$ (using the result derived from equation 61).

6.1.1 Radial and vertical temperature profiles and the condensation sequence

We first consider the thermal structure of the protoplanetary disc in the vertical direction given our previous discussion about flared discs. We have the isothermal sound speed given by

$$c_s^2(R) = \frac{\mathcal{R}}{\mu} T(R) \tag{136}$$

Table 2: Condensation temperatures for selected materials

T	Material
1680 K	Al_2O_3
1590 K	$CaTiO_3$
1400 K	$MgAl_2O_4$
1350 K	Mg_2SiO_4, iron alloys
370 K	Fe_3O_4
180 K	water ice
130 K	$NH_3 \cdot H_2O$
40 K – 80 K	methane, methane ices
50 K	argon

and from our previous discussion about vertical structure in accretion discs $H/R = c_s/v_k$ where v_k is the Keplerian velocity. A steady state disc viscously evolving disc has an effective temperature

$$T_{eff} = \left(\frac{3GM\dot{m}}{8\pi\sigma R^3}\right)^{1/4} \tag{137}$$

so we can write $T(R) = T_0 R^{-3/4}$. For the sound speed we have

$$c_s^2(R) = \frac{\mathcal{R}}{\mu} T_0 R^{-3/4} \tag{138}$$

which gives for the disc aspect ratio

$$\frac{H}{R} = \sqrt{\frac{\mathcal{R}T_0}{GM\mu}} R^{1/8}. \tag{139}$$

Thus we see that viscously heated discs are expected to flare slightly. This means that the disc at large radius will intercept a greater amount of radiation from the central star, which will have the effect of making the radial temperature profile somewhat shallower than the standard $R^{-3/4}$ power-law.

The protoplanetary disc will be composed of gas and dust grains whose mass ratio is expected to be $\simeq 100 : 1$. Planets are formed by the growth of the small dust grains up to planetary sized bodies. The variation of temperature within the protoplanetary disc as a function of distance from the central star allows various elements and compounds to condense into solids from the gas/vapour phase, so the local ratio of gas to solids will vary with distance to the central star. The temperature at which a chemical substance condenses into solid form is called the *condensation temperature*. Calculations of the abundance of solids in the disc requires the computation of the full equilibrium chemistry for a gas with a particular elemental abundance, at temperatures and densities (pressures) which are appropriate to protostellar discs. Substances with low condensation temperatures are usually referred to as ‘volatiles’, and those with high condensation temperatures are ‘refractories’. The table shows the condensation temperatures of some important compounds that can form either ice grains or dust grains.

The radial location of the disc midplane where the temperature drops below $\simeq 160$ K is called the 'snow line', as this is where water ices are expected to form. At this location the surface density of solids Σ_s increases by about a factor of 4 above its nominal value due to the formation of ice grains.

If we naively calculate the temperature in a disc using equation (104) using $\dot{m} = 10^{-8}$ M$_\odot$/yr we obtain 86K at 1 AU. Adopting $\dot{m} = 10^{-7}$ M$_\odot$/yr gives ~ 150 K at 1 AU. These estimates suggest that the snow line in the Solar System was at ≤ 1 AU, but meteoritic evidence suggests that the water content of asteroids increases out beyond about 3 AU. The original minimum mass solar nebula model has a snow line at 2.7 AU.

The effective temperature estimate we have used is really the temperature at the surface of the disc, since this is the temperature that the emitting surface of a black-body has as it emits radiation into free space. The midplane temperature is expected to be significantly higher. If the disc is optically thick in the vertical direction, and heat is transported from the midplane to the disc surface via radiation, then the flux is given by:

$$F(z) = -\frac{16\sigma T^3}{3\kappa_R \rho}\frac{dT}{dz} \tag{140}$$

where the optical depth from the midplane to surface is given by

$$\tau = \frac{1}{2}\kappa_R \Sigma$$

and κ_R is the Rosseland mean opacity. Let us assume that essentially all viscous dissipation occurs in the disc midplane ($z = 0$, i.e. this is where all disc heating occurs). Then the flux is constant with height. At the surface of the disc it is given by $F(z) = \sigma T_{eff}^4$, thus we can write

$$-\frac{16\sigma T^3}{3\kappa_R \rho}\frac{dT}{dz} = \sigma T_{eff}^4 \tag{141}$$

which can be integrated (assuming constant opacity) to give

$$-\frac{4\sigma}{3\kappa_R}\left[T^4\right]_{T_c}^{T_{eff}} = \sigma T_{eff}^4 \frac{\Sigma}{2}. \tag{142}$$

If we assume that the midplane temperature T_c is significantly higher that the effective temperature at the surface ($T_c >> T_{eff}$) then we obtain

$$T_c^4 = \frac{3\tau}{4} T_{eff}^4. \tag{143}$$

For most discs we expect $\tau \geq 100$ so that $T_c \simeq 3T_{eff}$. Thus we see that the location of the snow line can move out to larger radii, and this effect can be increased further by taking into account the absorption of radiation from the central star by the disc.