Physical Constants

Gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
Speed of light	c	3×10^8 m s^{-1}
Solar mass	$M_\odot$	2.0×10^{30}kg
Gravitational radius of Sun	$r_{g\odot}$	3 km
Hubble constant	H_0	70 km s^{-1}Mpc^{-1}
Hubble radius	c/H_0	6×10^3 Mpc

1 pc = 3.1×10^{16} m

NOTATION

Three-dimensional tensor indices are denoted by Greek letters $\alpha, \beta, \gamma, ...$ and take on the values 1, 2, 3.
Four-dimensional tensor indices are denoted by Latin letters $i, k, l, ...$ and take on the values 0, 1, 2, 3.
The metric signature $(+ - --)$ is used.
Partial derivatives are denoted by ",".
Covariant derivatives are denoted by ";".

USEFUL FORMULAS.

Cosmology

$$ds^2 = c^2dt^2 - R^2(t)\left[d\chi^2 + \frac{\sin^2(\sqrt{k}\chi)}{k}(d\theta^2 + \sin^2\theta d\phi^2)\right] \text{ (Robertson–Walker metric),}$$

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)R + \frac{\Lambda R}{3} \text{ (acceleration equation)}$$

$$q = -\frac{\ddot{R}R}{\dot{R}^2} \text{ deceleration parameter}$$

$$\dot{R}^2 + kc^2 = \frac{8\pi G}{3}\rho R^2 + \frac{\Lambda R^2}{3} \text{ (Friedmann equation)}$$

$$d(\rho c^2 V) = -pdV \text{ (energy conservation equation)}$$

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} = 0.92 \times 10^{-26} \text{ kg m}^{-3} \text{ (critical density)}$$

General Relativity

Minkowski metric:

$$ds^2 = \eta_{ik} dx^i dx^k = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

Covariant derivatives:

$$A^i_{;k} = A^i_{,k} + \Gamma^i_{km} A^m, \quad A_{i;k} = A_{i,k} - \Gamma^m_{ik} A_m, \text{ where } \Gamma^i_{kn} \text{ are Christoffel symbols}$$

Christoffel symbols:

$$\Gamma^i_{kl} = \frac{1}{2} g^{im} \left(g_{mk,l} + g_{ml,k} - g_{kl,m} \right)$$

Geodesic equation:

$$\frac{du^i}{ds} + \Gamma^i_{kn} u^k u^n = 0,$$

where

$$u^i = dx^i/ds \text{ is the 4-velocity along the geodesic.}$$

Riemann tensor:

$$A^i_{;k;l} - A^i_{;l;k} = -A^m R^i_{mkl}, \text{ where } R^i_{klm} = g^{in} R_{nklm},$$

$$R^i_{klm} = \Gamma^i_{km,l} - \Gamma^i_{kl,m} + \Gamma^i_{nl}\Gamma^n_{km} - \Gamma^i_{nm}\Gamma^n_{kl}.$$

Symmetry properties of the Riemann tensor:

$$R_{iklm} = -R_{kilm} = -R_{ikml}, \quad R_{iklm} = R_{lmik}.$$

Bianchi identity:

$$R^n_{ikl;m} + R^n_{imk;l} + R^n_{ilm;k} = 0.$$

Ricci tensor:

$$R_{ik} = g^{lm} R_{limk} = R^m_{imk}.$$

Scalar curvature:

$$R = g^{il} g^{km} R_{iklm} = g^{ik} R_{ik} = R^i_i.$$

Einstein equations:

$$R^i_k - \frac{1}{2}\delta^i_k R = \frac{8\pi G}{c^4} T^i_k,$$

where T^i_k is the Stress-Energy tensor.

Gravitational radius:

$$r_g = 2GM/c^2 = 3(M/M_\odot) \text{ km}, \text{ where } M_\odot \text{ is the mass of the Sun.}$$