Lecture 11. General Relativity is required

CONTENT

Newton's law has since been superseded by Einstein's theory of general relativity, but it continues to be used as an excellent approximation of the effects of gravity. Relativity is only required when there is a need for extreme precision, or when dealing with gravitation for very massive objects. The Universe is really very very massive object.

11.1. Disagreement of Newton's theory with observations

The predicted deflection of light by gravity using Newton's theory is only half the deflection actually observed. (Fig.11.1).

Newton's theory does not fully explain the precession of the perihelion of the orbits of the planets, especially of planet Mercury. (Fig.11.2).
There is a 43 arcsecond per century discrepancy between the Newtonian prediction and the observed precession.

The observed fact that gravitational and inertial masses are the same for all bodies is unexplained within Newton's system and treated within this theory just as coincidence.

11.2. Theoretical problems

There is no immediate prospect of identifying the mediator of gravity. Attempts by physicists to identify the relationship between the gravitational force and other known fundamental forces are not yet resolved.

Newton's theory requires that gravitational force is transmitted instantaneously. Newton himself felt the inexplicable action at a distance to be unsatisfactory. He was deeply uncomfortable with the notion of "action at a distance" which his equations implied.

In 1692 he wrote:

"That one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one another, is to me so great an absurdity that, I believe, no man who has in philosophic matters a competent faculty of thinking could ever fall into it."

11.3. Gravitational paradox in Newtonian theory

The main problem in application of Newton' theory to cosmology is related with the fact that the Universe can be infinite. When we consider gravitational forces generated by infinite Universe the result depends crucially on the way how we integrate gravitational forces from different mass elements of the Universe.

To illustrate this statement let us consider the so called shell theorem which says:

(i) A spherically symmetric body affects external objects gravitationally as though all of its mass were concentrated at a point at its center.

(ii) If the body is a spherically symmetric shell (i.e. a hollow ball), no gravitational force is exerted by the shell on any object inside, regardless of the object's location within the shell.

(iii) Inside a solid sphere of constant density the gravitational force varies linearly with distance from the center, becoming zero at the center of mass.

$$\mathbf{F} = \frac{\mathbf{GMm}}{\mathbf{r^2}} = \frac{\mathbf{4\pi G\rho mr^3}}{\mathbf{3r^2}} \propto \mathbf{r}. \tag{1}$$

(Fig.11.3) and (Fig.11.3a).

If we integrate over spheres then contribution of outside spheres is exactly equal to zero and only finite part of the Universe contributes to gravitational force. In this case the force is finite but arbitrary. (Fig.11.4).

If we divide the Universe into infinite planes we will conclude that all bodies in the Universe should experience infinite disruptive gravitational forces. This is in the direct contradiction with the fact of our existence. Even the direction of this disruption depends on the way of integration. (Fig.11.5).

Not going into detail of relevant calculations we can conclude that Newtonian theory when applied to the infinite Universe is not self-consistent. This inconsistency is called "Gravitational paradox" or the second Olber's paradox. [Let me remind you that the first Olber's paradox about darkness of the night sky was considered in Part I of this course.]

11.4. Inconsistency between Newton's theory and the accelerated expansion of the Universe

According to Newton's theory only matter density contributes into gravitational forces and the Universe should expand with deceleration. However, there is strong observational evidence that at present time our Universe is expanding with acceleration which implies some sort of repulsive gravitational forces. The substance generating such repulsive forces (anti-gravity) is the so called "dark energy". According to General relativity gravity is determined not only by density but by pressure and tension (which correspond to a negative pressure). Tensions contribute to gravitational acceleration with opposite sign in comparison with density. This is the reason why according to General relativity our Universe can expand with acceleration.

This is very brief motivation why we should study General Relativity (GR) when we are trying to understand the structure and evolution of the Universe. According to GR gravitation is an attribute of curved spacetime instead of being due to a force propagated between bodies. In Einstein's theory, masses distort spacetime in their vicinity, and other particles move in trajectories determined by the geometry of spacetime. This allowed a description of the motions of light and mass that was consistent with all available observations.