

Heavy Element Nucleosynthesis

A summary of the nucleosynthesis of light elements is as follows

${}^4\text{He}$	Helium burning
${}^3\text{He}$	Incomplete PP chain (H burning)
${}^2\text{H}$, Li, Be, B	Non-thermal processes (spallation)
${}^{14}\text{N}$, ${}^{13}\text{C}$, ${}^{15}\text{N}$, ${}^{17}\text{O}$	CNO processing
${}^{12}\text{C}$, ${}^{16}\text{O}$	Helium burning
${}^{18}\text{O}$, ${}^{22}\text{Ne}$	α captures on ${}^{14}\text{N}$ (He burning)
${}^{20}\text{Ne}$, Na, Mg, Al, ${}^{28}\text{Si}$	Partly from carbon burning
Mg, Al, Si, P, S	Partly from oxygen burning
Ar, Ca, Ti, Cr, Fe, Ni	Partly from silicon burning

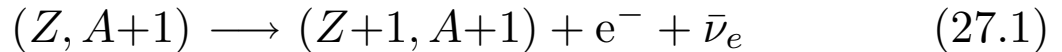
Isotopes heavier than iron (as well as some intermediate weight isotopes) are made through neutron captures. Recall that the probability for a non-resonant reaction contained two components: an exponential reflective of the quantum tunneling needed to overcome electrostatic repulsion, and an inverse energy dependence arising from the de Broglie wavelength of the particles. For neutron captures, there is no electrostatic repulsion, and, in complex nuclei, virtually all particle encounters involve resonances. As a result, neutron capture cross-sections are large, and are very nearly independent of energy.

To appreciate how heavy elements can be built up, we must first consider the lifetime of an isotope against neutron capture. If the cross-section for neutron capture is independent of energy, then the lifetime of the species will be

$$\tau_n = \frac{1}{N_n \langle \sigma v \rangle} \approx \frac{1}{N_n \langle \sigma \rangle v_T} = \frac{1}{N_n \langle \sigma \rangle} \left(\frac{\mu_n}{2kT} \right)^{1/2}$$

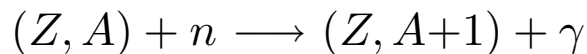
For a typical neutron cross-section of $\langle\sigma\rangle \sim 10^{-25}$ cm² and a temperature of 5×10^8 K, $\tau_n \sim 10^9/N_n$ years.

Next consider the stability of a neutron rich isotope. If the ratio of neutrons to protons in an atomic nucleus becomes too large, the nucleus becomes unstable to beta-decay, and a neutron is changed into a proton via



The timescale for this decay is typically on the order of hours, or $\sim 10^{-3}$ years (with a factor of $\sim 10^3$ scatter).

Now consider an environment where the neutron density is low ($N_n \sim 10^5$ cm⁻³), so that the lifetime of isotope Z, A against neutron capture is long $\sim 10^4$ years. In time, the isotope will capture a neutron, undergoing the reaction

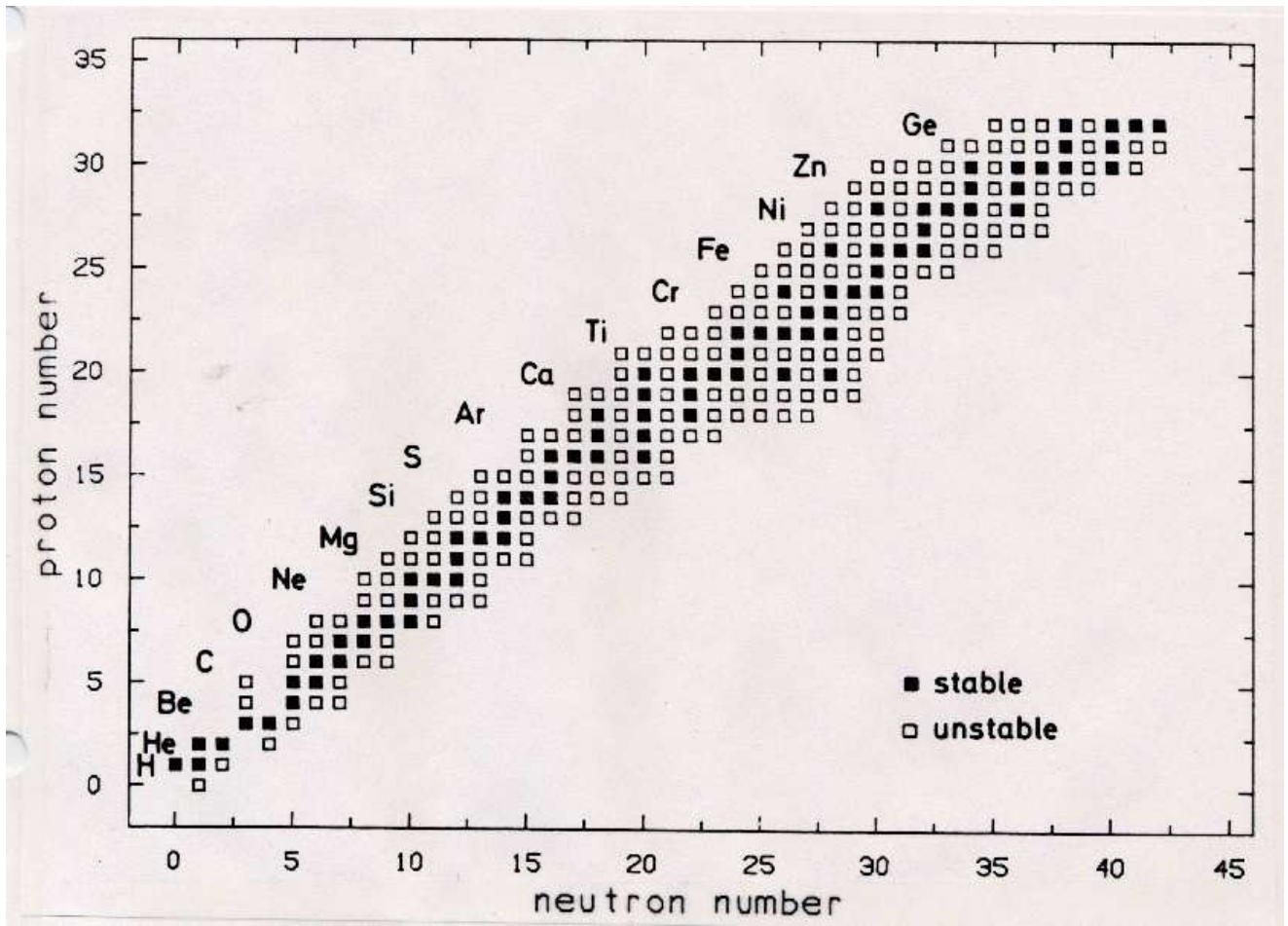


If isotope $Z, A+1$ is stable, then the process will repeat, and after a while, isotope $Z, A+2$ will be created. However, if isotope $Z, A+1$ is unstable to beta-decay, the decay will occur before the next neutron capture and the result will be isotope $Z+1, A+1$. This is the *s-process*; it occurs when the timescale for neutron capture is longer than the timescale for beta-decay.

Next, consider a different environment, where the neutron density is high, $N_n \sim 10^{23}$ cm⁻³. (This occurs during a supernova explosion.) At these neutron densities, the timescale for neutron capture is of the order of a millisecond, and isotope $Z, A+1$ will become $Z, A+2$ via a neutron capture before it can beta-decay. Isotope $Z, A+2$ (or its daughter isotope if it is radioactive) would therefore be an *r-process* element; its formation requires the rapid capture of a neutron.

Whether an isotope is an s -process or r -process element (or both) depends on both its properties and the properties of the isotopes surrounding it. In the example above, suppose the radioactive isotope $Z-3, A+1$ is formed during a phase of rapid neutron bombardment, and suppose that its daughters $Z-2, A+1$, $Z-1, A+1$, and $Z, A+1$ are all radioactive as well. Isotope $Z+1, A+1$ would then be the end product of the r -process, as well as the product of s -process capture and beta-decay from Z, A . On the other hand, if isotope $Z-1, A+1$ is stable, then the beta-decays from $Z-3, A+1$ will stop at $Z-1, A+1$, and element $Z+1, A+1$ would effectively be “shielded” from the r -process. In this case, element $Z+1, A+1$ would be an s -process isotope only. Isotope $Z-1, A+1$ would, of course, be an r -process element, and, if isotope $Z-2, A$ were unstable, it would be “shielded” from the s -process.

A third process for heavy element formation is the p -process. These isotopes are proton-rich and cannot be formed via neutron capture on any timescale. P -process isotopes exist to the left of the “valley of beta stability” and thus have no radioactive parent. These isotopes are rare, since their formation requires overcoming a coulomb barrier. Their formation probably occurs in an environment similar to that of the r -process, *i.e.*, where the proton density is extremely high, so that proton captures can occur faster than positron decays.



The valley of stability.

Isotopic Ratios

The abundance of every isotope A, Z can therefore be written

$$N(Z, A) = N_s(Z, A) + N_r(Z, A) + N_p(Z, A) \approx N_s(Z, A) + N_r(Z, A)$$

By knowing the neutron capture cross-sections of the isotopes, and by knowing which isotopes are shielded, (*i.e.*, which are solely s -process and which are solely r -process) the pattern of elemental abundances can be used to trace the history of all the species.

Clearly, the relative abundance of a given s -process element depends on its neutron capture cross-section. If the capture cross-section is low, then nuclei from the previous species will tend to “pile-up” at the element, and abundance of the isotope will be high. Conversely, if the isotope has a large neutron-capture cross-section, it will quickly be destroyed and will have a small abundance. S -process abundances can therefore be derived from differential equations of the form

$$\frac{dN_A}{dt} = -\sigma_A N_A + \sigma_{A-1} N_{A-1}$$

The starting point for these equations is ^{56}Ni (although neutron captures actually start during helium burning, since the reactions $^{13}\text{C}(\alpha, n)^{16}\text{O}$, $^{17}\text{O}(\alpha, n)^{20}\text{Ne}$, $^{21}\text{Ne}(\alpha, n)^{24}\text{Mg}$ and $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ all create free neutrons). The endpoint of the s -process is ^{209}Bi , since the next element in the series, ^{210}Bi α -decays back to ^{206}Pb . Thus, the entire network looks like

$$\frac{dN_{56}}{dt} = -\sigma_{56} N_{56}$$

$$\frac{dN_A}{dt} = -\sigma_A N_A + \sigma_{A-1} N_{A-1} \quad 57 \leq A \leq 209; A \neq 206$$

$$\frac{dN_{206}}{dt} = -\sigma_{206} N_{206} + \sigma_{205} N_{205} + \sigma_{209} N_{209} \quad (27.2)$$

with the boundary conditions

$$N_A(t = 0) = \begin{cases} N_{56}(t = 0) & A = 56 \\ 0 & A > 56 \end{cases}$$

These equations can be simplified *locally* by noting that the neutron capture cross-sections for adjacent non-closed-shell nuclei are large and of the same order of magnitude, so that

$$\sigma_A N_A \approx \sigma_{A-1} N_{A-1} \quad (27.3)$$

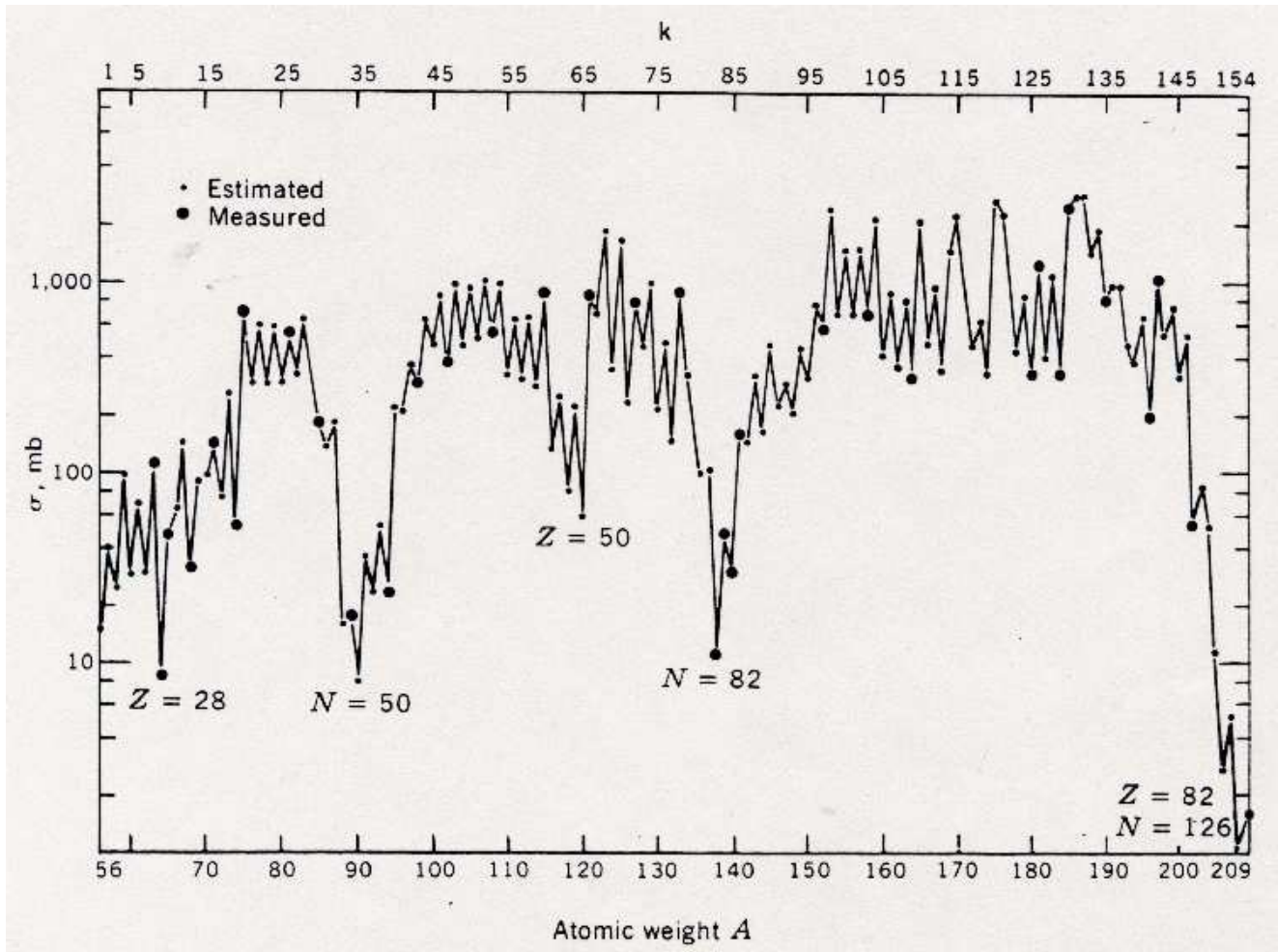
This is called the *local approximation*; it is only good for some adjacent nuclei. Over a length of $A \sim 100$, abundances can drop by a factor of ~ 10 .

The abundance of *r*-process isotopic ratios in nature can be estimated by differencing the observed cosmic abundances from that predicted from the *s*-process. Part of the *r*-process isotopic ratios may be due to processes associated with nuclear statistical equilibrium under extremely high densities ($\rho > 10^{10}$ gm-cm⁻³). (Under these conditions, the Fermi exclusion will force electron captures in the nucleus, and cause a build up a neutrons.) However, *r*-process isotopes can also be built up in more moderate densities, if the neutron density is high ($\sim 10^{23}$ cm⁻³). Under these conditions, a nucleus will continue to absorb neutrons until it can capture no more (*i.e.*, until another neutron would exceed the binding energy). At this point, the nucleus must wait until a beta-decay occurs. Thus, the abundance of an isotope with charge Z will be governed by the beta-decay rates, *i.e.*,

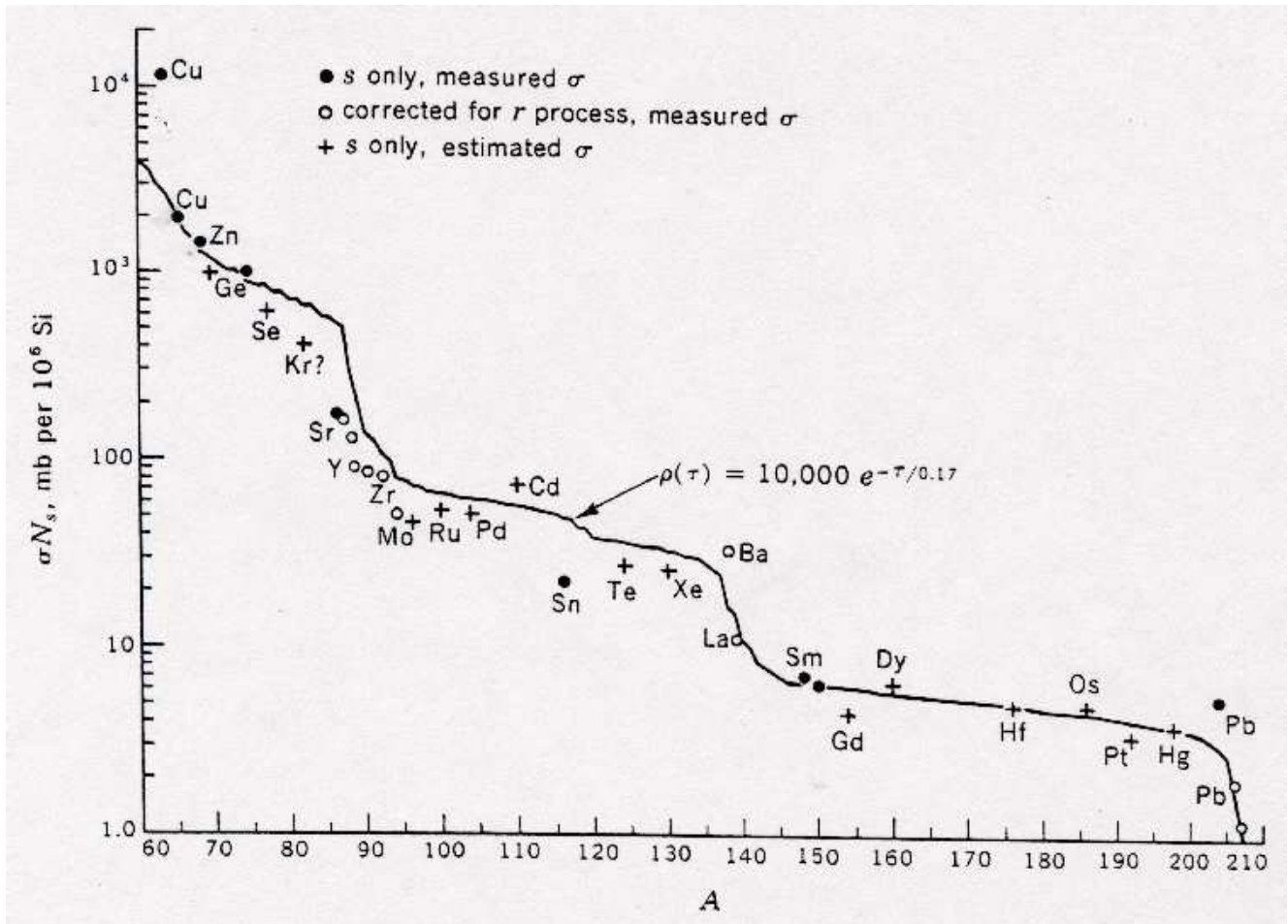
$$\frac{dN_Z}{dt} = \lambda_{Z-1} N_{Z-1}(t) - \lambda_Z N_Z(t)$$

where λ_Z is the beta-decay rate at the waiting point for charge Z . (In practice, under these circumstances, one must also take into

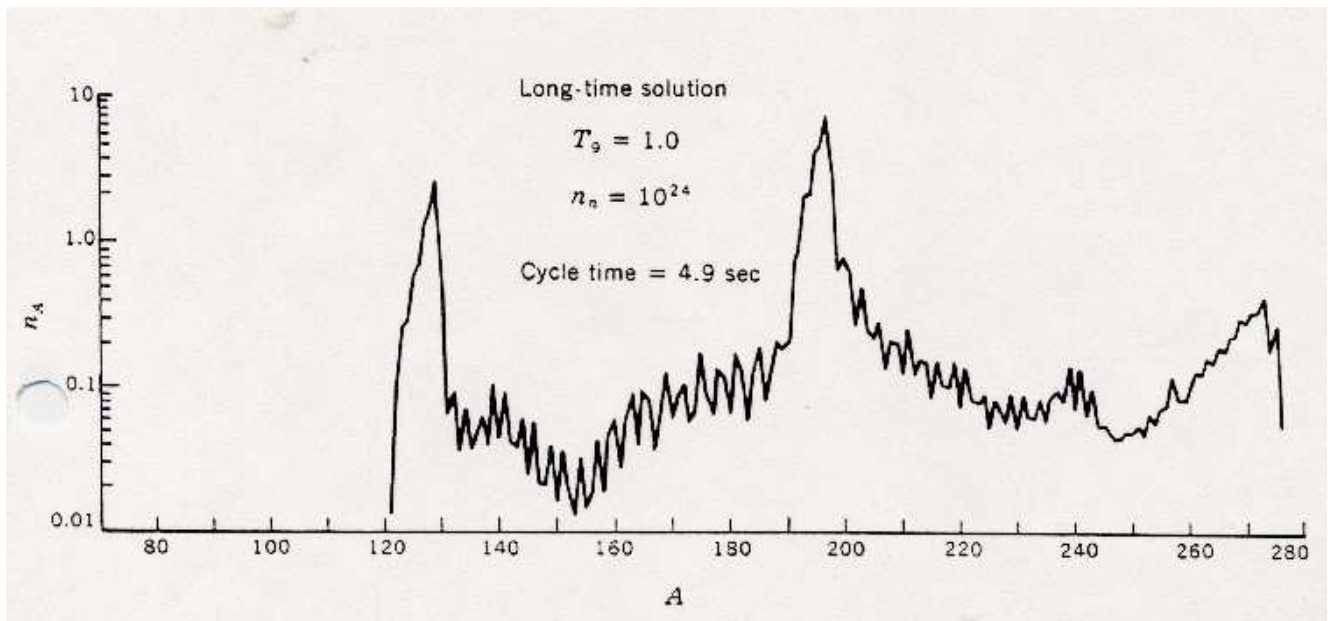
account proton and α -particle captures as well, since these reactions will also add charge to neutron-rich material. However, because of the electrostatic repulsion, these capture cross-sections will be much lower.) *R*-process nucleosynthesis stops at $Z = 94$ ($N = 175$) where nuclear fission occurs to return two additional seeds to the capture chain.



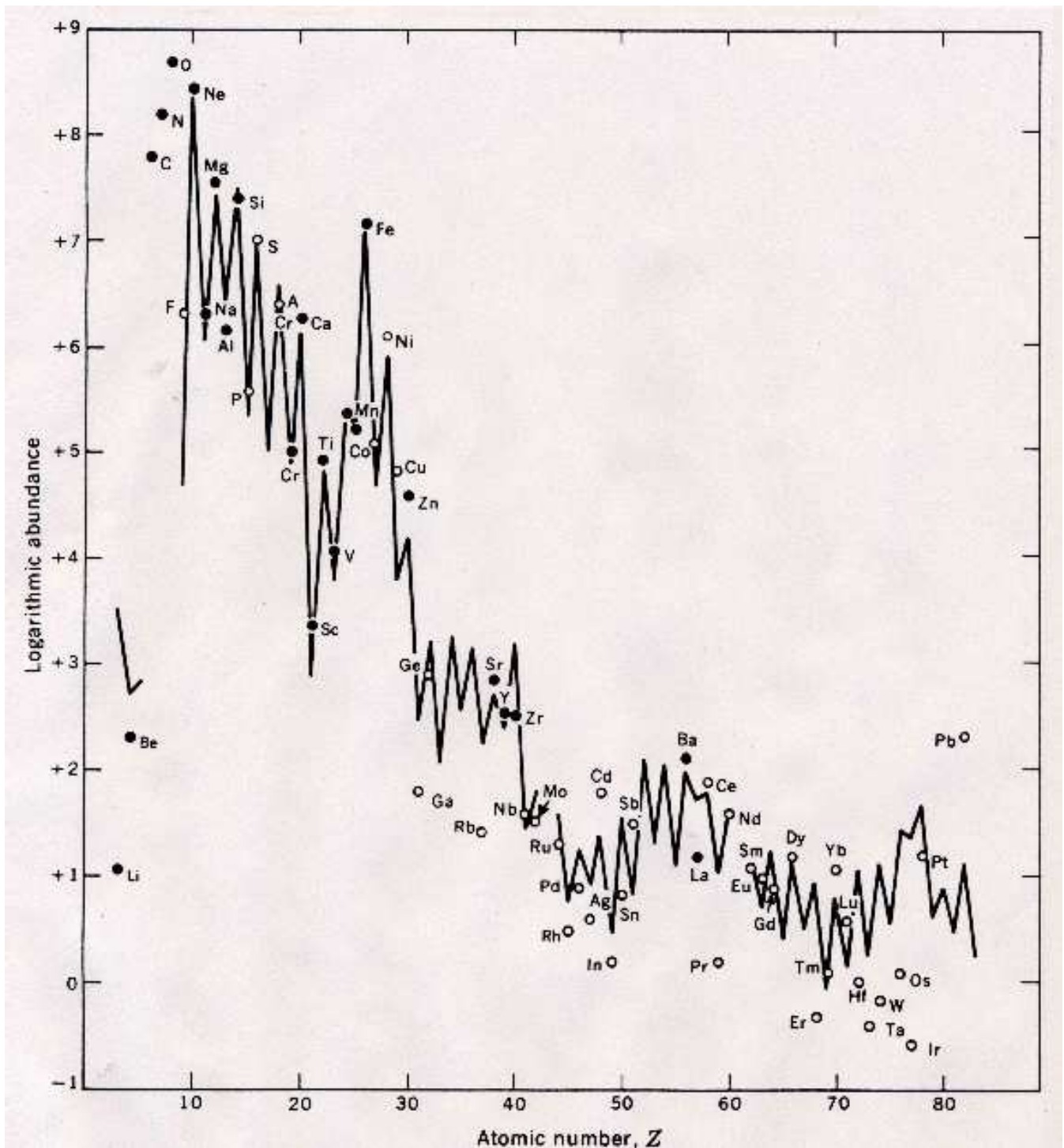
Measured and estimated neutron-capture cross sections of nuclei on the *s*-process path. The neutron energy is near 25 keV. The cross sections show a strong odd-even effect, reflecting average level densities in the compound nucleus. Even more obvious is the strong influence of closed nuclear shells, or magic numbers, which are associated with precipitous drops in the cross section. Nucleosynthesis of the *s*-process is dominated by the small cross sections of the neutron-magic nuclei.



The solar system σN_s curve, *i.e.*, the product of the neutron capture cross sections for $kT = 30$ keV times the nuclide abundance per 10^6 silicon atoms. The solid curve is the calculated result of an exponential distribution of neutron exposures.



Calculated r -process abundances after long times. The nuclei have been driven around a fission cycle until the abundance distribution becomes characteristic of a steady state. The abundance of each nucleus grows exponentially with an e-folding time of 4.9 sec. This time, calculated on the basis of β -decays rates, depends upon the neutron density and the the temperature.



The abundances of the elements in the solar system. The dots represent values obtained from the strengths of solar absorption lines, while the line is based on chemical evidence from the earth and meteorites.

Nucleocosmochronology

It is possible to obtain an independent estimate of the age of the Solar System (and, indeed the universe) from the study of radioactive species (and sometimes their daughter elements). The idea is simple. Consider an radioactive species, i present in an object that was born at time $\tau = t$ in the universe. Prior to time t , this element was created in various events, and had a production rate $P_i p(t)$. The abundance of that species today (at time $t = t_0$) will be

$$N = P_i e^{-\lambda_i(t_0-t)} \int_0^t p(\tau) e^{-\lambda_i(t-\tau)} d\tau$$

which simplifies to

$$N = P_i e^{-\lambda_i t_0} \int_0^t p(\tau) e^{\lambda_i \tau} d\tau \quad (27.4)$$

where λ is the decay rate of the species. Obviously, calculations of the production rate can be treacherous. However, if one takes the ratio of two elements which are presumed to be created together (say, in r -process events), then most of the unknowns cancel out. So

$$\frac{N_i}{N_s} = \frac{P_i}{P_s} \frac{e^{-\lambda_i t_0} \int_0^t p(\tau) e^{\lambda_i \tau} d\tau}{e^{-\lambda_s t_0} \int_0^t p(\tau) e^{\lambda_s \tau} d\tau}$$

Moreover, if the second element is stable (with $\lambda_s = 0$), then

$$\begin{aligned} \frac{N_i}{N_s} &= \frac{P_i}{P_s} e^{-\lambda_i t_0} \frac{\int_0^t p(\tau) e^{\lambda_i \tau} d\tau}{\int_0^t p(\tau) d\tau} \\ &\approx \frac{P_i}{P_s} (1 - \lambda_i(t_0 - \langle t \rangle)) \end{aligned} \quad (27.5)$$

In other words, by observing the ratio of two species, and knowing the ratio of their production ratio in a star, one can obtain $t_0 - \langle t \rangle$, the mean age of the elements.

Obviously to do this experiment, one needs to work with elements that are formed in the same event. In the solar system, one can restrict the experiment to r -process only isotopes. However, it is not possible to separate out the different isotopes via stellar absorption lines. (This is not strictly true for the lightest species such as deuterium and CNO, but for heavier elements it is certainly true.) However, elements heavier than ^{209}Bi can only be made with the r -process (since ^{210}Bi α -decays back to ^{206}Pb).

The two most popular elements to perform this experiment with is Uranium and ^{232}Th (half-life 13.9 Gyr) and ^{238}U (half-life 4.51 Gyr). But these lines are weak!) For example, here are the results from the extreme metal-poor halo star BD+17 3248 (from Cowan *et al.* 2002)

Chronometric Age Estimates for BD+17° 3248

Element Pair	Predicted	Observed	Solar	Age (Gyr)	
				Best	Limit
Th/Eu	0.507	0.309	0.4614	10.0	> 8.2
Th/Ir	0.0909	0.03113	0.0646	21.7	> 14.8
Th/Pt	0.0234	0.0141	0.0323	10.3	> 16.8
Th/U	1.805	7.413	2.32	> 13.4	> 11.0
U/Ir	0.05036	0.0045	0.0369	> 15.5	> 13.5
U/Pt	0.013	0.0019	0.01846	> 12.4	> 14.6