UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **PHAS4317**

MODULE NAME : **Galaxy and Cluster Dynamics**

DATE : **21-May-07**

TIME : **10:00**

TIME ALLOWED : **2 Hours 30 Minutes**

2006/07-PHAS4317B-001-EXAM-13

TURN OVER

Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Solar radius	$R_\odot$	=	6.96×10^{8} m
Solar mass	$M_\odot$	=	2.0×10^{30} kg
Solar Luminosity	$L_\odot$	=	3.83×10^{26} J s^{-1}
Parsec	pc	=	3.09×10^{16} m
Gravitational constant	G	=	6.67×10^{-11} N m^2 kg^{-2}
Distance to Galactic Centre	R_0	=	10 kpc
Sun's orbital circular velocity	v_0	=	250 kms^{-1}
Oort's constants	A	=	15 kms^{-1} kpc^{-1}
	B	=	–10 kms^{-1} kpc^{-1}

In spherical co-ordinates:

$$\nabla^2 F = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial F}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial F}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 F}{\partial\phi^2}$$

1. (a) Define the *distribution function* of stars in phase space. Describe the difference between *isolating* and *non-isolating* integrals with the help of an example, and state *Jeans Theorem.* [5]

(b) State how the density (ρ) and the gravitational potential (V) are related in a self-gravitating system. Define your terms. [2]

(c) Show that the (spherically symmetric) density distribution:

$$\rho(r) = \left(\frac{M}{4\pi r_0^3}\right) \frac{r_0^4}{r^2(r+r_0)^2}$$

is consistent with the potential

$$V(r) = \frac{GM}{r_0} \ln\left(\frac{r}{r+r_0}\right)$$

where r_0 and M are constants. [7]

(d) Show that M is the total mass of this system. Derive and sketch the radial dependence of the circular rotation speed, $v_{\mathrm{circ.}}(r)$, paying particular attention to the cases where $r << r_0$ and $r >> r_0$. [6]

2. (a) Write down the general equation of the virial theorem, defining the terms. [4]

(b) Write down the total kinetic energy and the total potential energy of a system of stars in terms of the mass-weighted mean square velocity, $\langle v^2 \rangle$, its total mass, M, and its size, R. Hence use the virial theorem to define a *virial mass* for the system. [3]

(c) The two-body relaxation time for a stellar system is given by:

$$t_{relax} = \frac{1}{16}\sqrt{\frac{3}{\pi}} \cdot \frac{\langle v_1^2 \rangle^{3/2}}{n(Gm_2)^2 ln[D_0 v^2/G(m_1 + m_2)]}$$

Define the terms in this expression. Assuming that $v^2 = 2\langle v_1^2 \rangle$, $D_0 = R$, that the average stellar mass is $m(= m_1 = m_2)$ and that the distribution of stars is uniform within the stellar system, use the virial theorem to write down an expression for t_{relax} in terms of N (the number of stars), R, G and m. [6]

(d) What is the crossing time, t_{cross} for a star cluster (in terms of R and $\langle v^2 \rangle$)? Show that

$$\frac{t_{relax}}{t_{cross}} \sim \sqrt{\frac{\pi}{3}} \frac{N}{32 ln(N/2)}$$

[3]

(e) Do you expect galactic clusters to be (i) relaxed, (ii) well described by the Collisionless Boltzmann Equation? What about globular clusters? Give reasons for your answer. [4]

3. (a) Define the terms *kinematic* and *dynamical local standards of rest* in the Galaxy. What is the *Stromberg asymmetric drift*? Explain (in simple terms) its physical origin. [7]

(b) Sketch the rotation curve of the Galaxy. Label your axes and provide appropriate axis scales. Give one reason why we cannot yet use the rotation curve of the Galaxy alone to deduce the density distribution within the Galaxy. [7]

(c) The z component of acceleration at a distance z above an infinite uniform disk of density ρ_0 is $k_z = -4\pi G\rho_0 z$. Derive an expression for the time it takes to get from maximum height above to maximum distance below the disk. [6]

4. (a) Describe how gravitational encounters may lead to *evaporation* or *ejection* from a stellar system, distinguishing carefully between the two processes. What are the names of the mathematical descriptions appropriate for the two cases? [6]

(b) Consider a cluster that is *evaporating*. Write down an expression for the total energy of the cluster as a function of its mass, M, and radius, R. Assuming that a cluster loses a fraction, k_{esc} of its mass per median relaxation time, t_{rel}, given by

$$t_{rel} \sim t^0_{rel}\sqrt{\frac{MR^3}{M_0 R_0^3}}$$

where M_0, R_0 and t^0_{rel} are the initial mass, radius and relaxation timescale respectively, use these results to write down an expression for (dM/dt) as a function of M and hence show that $M(t)$ is given by:

$$M(t) = M_0\left(1 - \frac{7k_{esc}t}{2t^0_{rel}}\right)^{2/7}$$

[8]

(c) How does the density of the core, $M(t)$, depend on its mass? Explain why the cluster has a halo, and what happens to it? Describe how *mass-segregation* acts to accelerate the process of core collapse. Explain how binary stars can affect core collapse, distinguishing between 'hard' and 'soft' binaries. [6]

5.(a) Using the Oort model of Galactic rotation show with the aid of a diagram that the radial and tangential velocities of a star are given by $v_{rad} = A\,d \sin 2l$ and $v_{tan} = A\,d \cos 2l + Bd$ respectively, where A and B are Oort's Constants. Show further that

$$A + B = -\left(\frac{dv_c}{d\varpi}\right)_{R_o} \quad \text{and} \quad A - B = \frac{v_c}{R_o},$$

State all assumptions and define all terms. [12]

You may use the relationship

$$\left(\frac{d\dot{\theta}}{d\varpi}\right)_{R_o} = \frac{1}{R_o}\left(\frac{d\Theta}{d\varpi}\right)_{R_o} - \frac{\Theta_o}{R_o^2}.$$

(b) The Jeans length and critical rotation length for a flat disk are approximately given by

$$L_J = \frac{\pi}{8}\frac{\langle v^2\rangle}{G\mu}$$

and

$$L_{rot} = \frac{2\pi G\mu}{3B^2}.$$

Explain what these length-scales represent and what is meant by the Toomre criterion. [4]

(c) A region of the Galaxy has a surface density, μ, of 40 $M_{\odot}pc^{-2}$ and a velocity dispersion of $\langle v^2\rangle^{1/2}$= 25 kms^{-1}. What does this imply about the stability of the disk in that region? [4]